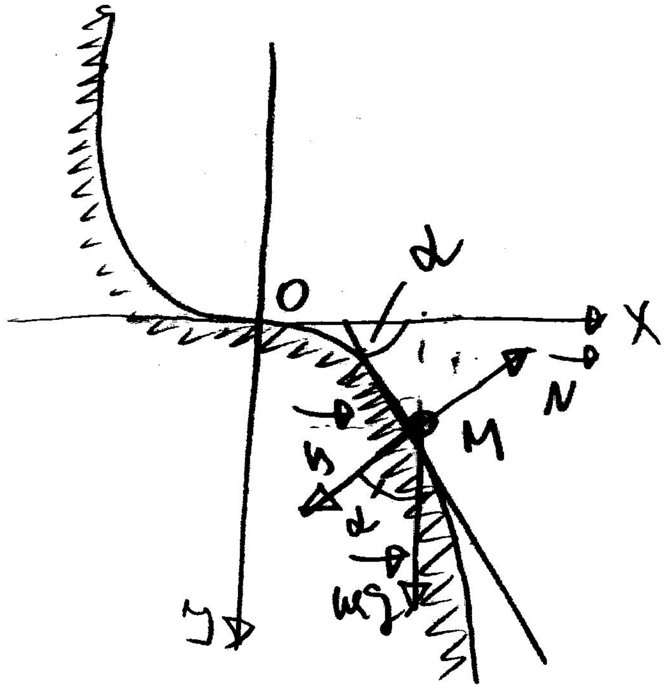
3.36 $y = \frac{x^3}{a^2}$

m

$V_0 = 0$

$x_1 - y_1 ?$



$T - \underbrace{T_0}_{0} = A(m\vec{g})$

$\frac{1}{2}\cancel{m}V^2 = \cancel{m}g\,y$

$$\boxed{V^2 = 2gy = 2g\frac{x^3}{a^2}}$$

$m\vec{a} = m\vec{g} + \vec{N}$

$n:\ m\frac{V^2}{R_k} = -N + mg\cos\alpha$

$y' = 3\frac{x^2}{a^2}$

$R_k = \frac{(1+y'^2)^{3/2}}{|y''|}$

$y'' = 6\frac{x}{a^2}$

$$R_k = \frac{\left(1+\frac{9}{a^4}x^4\right)^{3/2}}{\frac{6}{a^2}|x|}$$

$\operatorname{tg}\alpha = y' = \frac{3x^2}{a^2}$

$\cos\alpha = \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1+\frac{9x^4}{a^4}}}$

Услов.: $N = 0$

$$\frac{\dfrac{2\cancel{\gamma}\,\dfrac{x_1^3}{a^2}}{\left(1+\dfrac{9}{a^4}x_1^4\right)^{3/2}}}{\dfrac{6}{a^2}|x_1|} = \cancel{\gamma}\,\frac{1}{\left(1+\dfrac{9x_1^4}{a^4}\right)^{1/2}} \quad \Big/ \cdot \left(1+\frac{9x_1^4}{a^4}\right)^{1/2}$$

$$\frac{2\,\dfrac{x_1^3}{a^2}\,\dfrac{6}{a^2}|x_1|}{\left(1+\dfrac{9}{a^4}x_1^4\right)} = 1$$

$$\frac{12}{a^4}x_1^4 = 1+\frac{9}{a^4}x_1^4$$

$$\frac{3}{a^4}x_1^4 = 1 \Rightarrow x_1^4 = \frac{a^4}{3} \Rightarrow \boxed{x_1 = \frac{a}{\sqrt[4]{3}}}$$

$$\boxed{y_1 = \frac{1}{a^2}x_1^3} = \frac{1}{\cancel{a^2}}\,\frac{a^{\cancel{3}}}{3^{\frac{3}{4}}} = \boxed{\frac{a}{\sqrt[4]{27}}}$$

3.42

m

$y = \dfrac{4x^2}{a}$, $a > 0$

$M_0(0,0)$

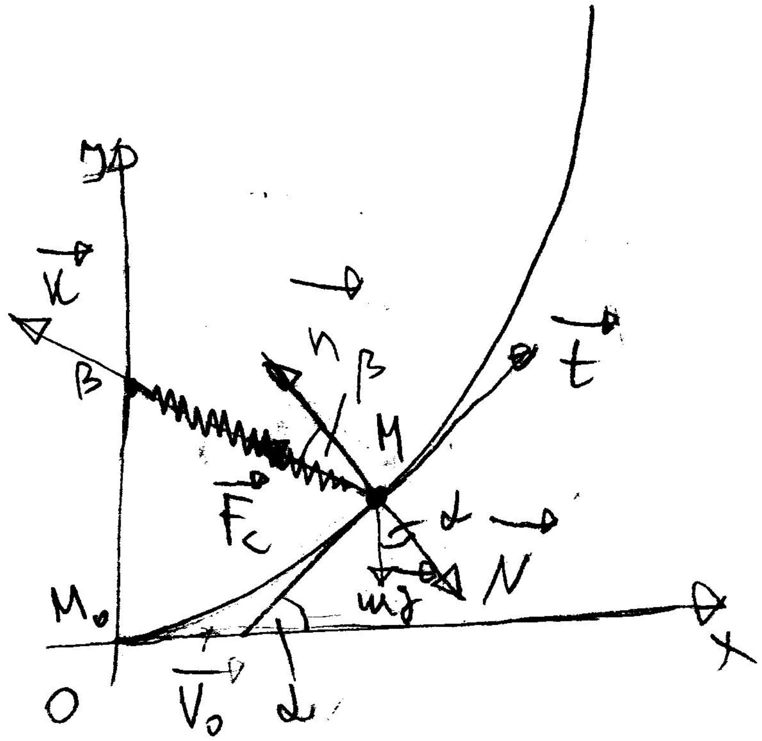
$V_0 = \sqrt{ag}$

$c = \dfrac{4mg}{a}$

$B(0, a)$

$l_0 = \dfrac{a}{4}$ $A\left(\dfrac{a}{2}, y_A\right)$

$N_A - ?$



$y_A = \dfrac{\cancel{4}}{\cancel{a}} \dfrac{a^{\cancel{2}}}{\cancel{4}} = a$

$A\left(\dfrac{a}{2}, a\right)$

$T_1 - T_0 = A_{0,1}(m\vec{g}) + A_{0,1}(\vec{F_c})$

$\dfrac{1}{2} m (V_1^2 - V_0^2) = -mg y_A + \dfrac{1}{2} c \left[(r_1 - r_0)^2 - (r_2 - r_0)^2 \right] \Big| \cdot \dfrac{2}{m}$

$V_1^2 = V_0^2 - 2ga + \dfrac{c}{m} \left[\left(a - \dfrac{a}{4}\right)^2 - \left(\dfrac{a}{2} - \dfrac{a}{4}\right)^2 \right]$

$r_2 = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{\left(\dfrac{a}{2} - 0\right)^2 + \underbrace{(a - a)^2}_{0}} = \dfrac{a}{2}$

$\boxed{V_1^2} = ag - 2ga + \dfrac{c}{m} \left(\dfrac{9}{16} a^2 - \dfrac{a^2}{16} \right) =$

$= -ag + \dfrac{c}{m} \dfrac{a^2}{2} = -ag + \dfrac{a^{\cancel{2}}}{\cancel{2m}} \dfrac{\cancel{4m}g}{\cancel{a}} = \boxed{ag}$

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_c$$

$$n: \; m\frac{v_1^2}{\rho_{k1}} = -N_1 - mg\cos\alpha_1 + -c(r_2 - r_0)\cos\beta_1$$

$$y' = \frac{8}{a}x \qquad y'_1 = \frac{8}{a}\frac{a}{2} = 4$$

$$y'' = \frac{8}{a}$$

$$\rho_{k1} = \frac{(1+y_1'^2)^{3/2}}{|y''_1|} = \frac{(1+16)^{3/2}}{\frac{8}{a}} = \frac{a}{8}17\sqrt{17};$$

$$\cos\alpha_1 = \frac{1}{\sqrt{1+tg^2\alpha_1}} = \frac{1}{\sqrt{1+16}} = \frac{1}{\sqrt{17}}; \sin\alpha_1 = \frac{4}{\sqrt{17}}$$

$$\vec{n}_1 = \vec{i}\cos\left(\alpha_1 + \frac{\pi}{2}\right) + \vec{j}\sin\left(\alpha_1 + \frac{\pi}{2}\right) =$$

$$= -\vec{i}\sin\alpha_1 + \vec{j}\cos\alpha_1 = -\frac{4}{\sqrt{17}}\vec{i} + \frac{1}{\sqrt{17}}\vec{j}$$

$$\vec{K}_1 = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} = \left(0 - \frac{a}{2}\right)\vec{i} + (a - a)\vec{j}$$

$$\vec{K}_1 = -\frac{a}{2}\vec{i}, \; |\vec{K}_1| = \frac{a}{2}$$

$$\vec{K}_{01} = \frac{\vec{K}_1}{|\vec{K}_1|} = -\vec{i}$$

$$\vec{n}_1 \cdot \vec{K}_{01} = |\vec{n}_1|\,|\vec{K}_{01}|\cos\beta_1$$

$$\left(-\frac{4}{\sqrt{17}}\vec{i} + \frac{1}{\sqrt{17}}\vec{j}\right)\cdot\left(-\vec{i}\right) = \frac{4}{\sqrt{17}} = \cos\beta_1$$

$$\boxed{N_1} = -\frac{mg}{\sqrt{17}} + \frac{4mg}{a}\left(\frac{a}{2}-\frac{a}{4}\right)\frac{4}{\sqrt{17}} - \frac{mg\cancel{a}}{\frac{\cancel{a}}{8}\,17\sqrt{17}}$$

$$N_1 = mg\left(-\frac{1}{\sqrt{17}} + \frac{4}{\sqrt{17}} - \frac{8}{17\sqrt{17}}\right) =$$

$$= mg\,\frac{-17+4\cdot 17-8}{17\sqrt{17}} = \boxed{\frac{43\,mg}{17\sqrt{17}}}$$

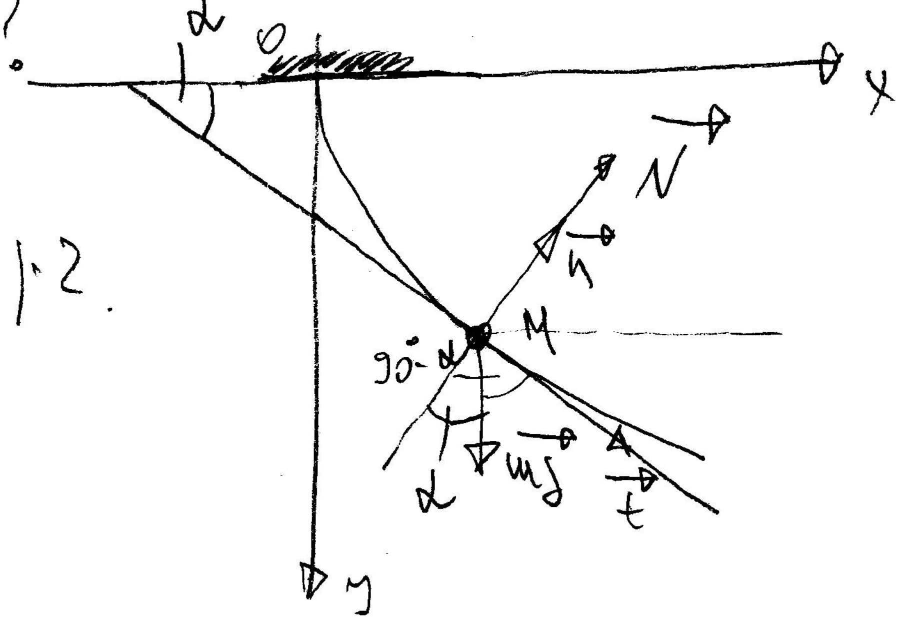
3.44 m

$x = R(\varphi - \sin\varphi)$

$y = R(1-\cos\varphi)$

$t_0 = 0 \quad M_0(0,0) \quad V_0 = 0$

$N(t)$ - ?



$T - T_0 = A(m\vec{g})$

$\frac{1}{2}\cancel{m}(V^2 - \underbrace{V_0^2}_{0}) = \cancel{m}gy \quad |\cdot 2$

$$\boxed{V^2 = 2gy}$$

$m\vec{a} = m\vec{g} + \vec{N}$

$n:\ m\frac{V^2}{\rho} = -mg\cos\alpha + N$

$$N = mg\left(\cos\alpha + \frac{2y}{\rho}\right)$$

$$\dot{x} = R(\dot{\varphi} - \dot{\varphi}\cos\varphi) = R\dot{\varphi}(1-\cos\varphi)$$

$$\dot{y} = R\dot{\varphi}\sin\varphi$$

$$v^2 = 2gy = 2gR(1-\cos\varphi) = R^2\dot{\varphi}^2(1 - 2\cos\varphi + \cos^2\varphi + \sin^2\varphi) =$$

$$= R^2\dot{\varphi}^2(2 - 2\cos\varphi) = \cancel{2}R^{\cancel{2}}\dot{\varphi}^2(\cancel{1-\cos\varphi}) = \cancel{2}g\cancel{R}(\cancel{1-\cos\varphi})$$

$$R\dot{\varphi}^2 = g \Rightarrow \dot{\varphi}^2 = \frac{g}{R} \Rightarrow \dot{\varphi} = \sqrt{\frac{g}{R}}$$

$$\boxed{\varphi} = \underbrace{\varphi_0}_{=0} + \sqrt{\frac{g}{R}}\,t = \boxed{\sqrt{\frac{g}{R}}\,t}$$

$t_0 = 0 \quad x_0 = 0, \quad y_0 = 0 \quad$ за $\varphi_0 = 0$

$$\rho_K = \frac{v^2}{\sqrt{a^2 - a_T^2}} = \frac{v^2}{a_N} = \frac{v^3}{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}$$

$$v^2 = 2gR(1-\cos\varphi) = 2gR\left(1 - \cos\sqrt{\frac{g}{R}}\,t\right)$$

$$\dot x = R\sqrt{\frac{g}{R}}\left(1-\cos\sqrt{\frac{g}{R}}\,t\right) = \sqrt{Rg}\left(1-\cos\sqrt{\frac{g}{R}}\,t\right)$$

$$\dot y = R\sqrt{\frac{g}{R}}\sin\sqrt{\frac{g}{R}}\,t = \sqrt{Rg}\sin\sqrt{\frac{g}{R}}\,t$$

$$\ddot x = g\sin\sqrt{\frac{g}{R}}\,t$$

$$\ddot y = g\cos\sqrt{\frac{g}{R}}\,t$$

$$|\dot x\ddot y - \dot y\ddot x| = g\sqrt{Rg}\,|\cos\varphi - \cos^2\varphi - \sin^2\varphi| =$$

$$= g\sqrt{Rg}\,|\cos\varphi - 1| = g\sqrt{Rg}\,(1-\cos\varphi) =$$

$$\boxed{\rho} = \frac{v^3}{|\dot x\ddot y - \dot y\ddot x|} = \frac{\left(2gR(1-\cos\varphi)\right)^{3/2}}{g\sqrt{Rg}\,(1-\cos\varphi)} =$$

$$= \frac{2\cancel{g}R\,\cancel{(1-\cos\varphi)}\sqrt{2\cancel{gR}(1-\cos\varphi)}}{\cancel{g}\sqrt{\cancel{Rg}}\,\cancel{(1-\cos\varphi)}} =$$

$$= 2R\sqrt{2(1-\cos\varphi)} = 2R\cdot 2\sin\frac{\varphi}{2} =$$

$$= \boxed{4R\sin\frac{1}{2}\sqrt{\frac{g}{R}}\,t}$$

$$\frac{dy}{dx} = \frac{\cancel{2}\cancel{R}(1-\cos\varphi)}{\cancel{2}\sqrt{2}\,\cancel{R}\sqrt{1-\cos\varphi}} = \frac{1}{\sqrt{2}}\sqrt{(1-\cos\varphi)} =$$

$$= \sin\frac{\varphi}{2} = \sin\frac{1}{2}\sqrt{\frac{g}{R}}\,t$$

$$\operatorname{tg}\alpha = y' = \frac{dy}{dx}\,\frac{dt}{dt} = \frac{\dot{y}}{\dot{x}} = \frac{\sin\varphi}{1-\cos\varphi} =$$

$$= \frac{\sin\varphi}{2\sin^2\frac{\varphi}{2}} = \frac{\cancel{2}\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}{\cancel{2}\sin^{\cancel{2}}\frac{\varphi}{2}} = \frac{\cos\frac{\varphi}{2}}{\sin\frac{\varphi}{2}} =$$

$$= \operatorname{ctg}\frac{\varphi}{2}$$

$$\cos\alpha = \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1+\operatorname{ctg}^2\frac{\varphi}{2}}} = \sin\frac{\varphi}{2}$$

$$N = mg\left(\sin\frac{\varphi}{2}+\sin\frac{\varphi}{2}\right) = 2mg\sin\frac{\varphi}{2} =$$

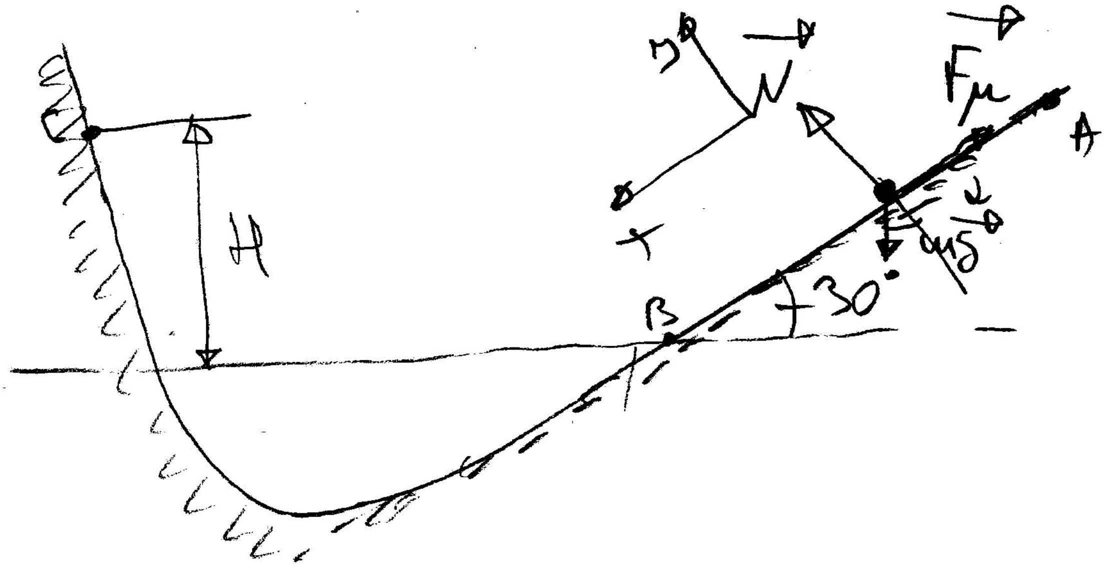
$$= 2mg\sin\frac{1}{2}\sqrt{\frac{g}{R}}\,t$$

3.47 $\alpha = 30°$

μ

$T, V_0 = 0$

$H - ?$



$$\vec{K}_B - \vec{K}_A = \int_0^T (m\vec{g} + \vec{N} + \vec{F}_\mu)\, dt$$

$$x: \cancel{m}(V_B - 0) = (\cancel{m}g\sin\alpha - \cancel{m}g\cos\alpha\,\mu)\,T$$

$$V_B = gT(\sin\alpha - \mu\cos\alpha)$$

$$\underbrace{T_C}_{=0} - T_B = A_{B,C}(m\vec{g})$$

$$-\frac{1}{2}\cancel{m}V_B^2 = -\cancel{m}gH$$

$$\frac{1}{2}g^{\cancel{2}}T^2(\sin\alpha - \mu\cos\alpha)^2 = \cancel{g}H$$

$$\boxed{H} = \frac{1}{2}gT^2(\sin\alpha - \mu\cos\alpha)^2 =$$

$$= \frac{1}{2}gT^2\left(\frac{1}{2} - \mu\frac{\sqrt{3}}{2}\right)^2 = \boxed{\frac{gT^2}{8}(1 - \mu\sqrt{3})^2}$$